



Skew group algebras of piecewise hereditary algebras are piecewise hereditary

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ARTICLE INFO

Article history:

Received 27 April 2007

Received in revised form 2 May 2008

Available online 26 July 2008

Communicated by I. Reiten

MSC:

16S35

18E30

ABSTRACT

We show that the main results of Happel–Rickard–Schofield (1988) and Happel–Reiten–Smalø (1996) on piecewise hereditary algebras are coherent with the notion of group action on an algebra. Then, we take advantage of this compatibility and show that if G is a finite group acting on a piecewise hereditary algebra A over an algebraically closed field whose characteristic does not divide the order of G , then the resulting skew group algebra $A[G]$ is also piecewise hereditary.

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Let k be an algebraically closed field. For a finite dimensional k -algebra A , we denote by $\text{mod } A$ the category of finite dimensional left A -modules, and by $D^b(A)$ the (triangulated) derived category of bounded complexes over $\text{mod } A$. Let \mathcal{H} be a connected hereditary abelian k -category. Following [7] (compare [5,9]), we say that A is *piecewise hereditary of type \mathcal{H}* if it is derived equivalent to \mathcal{H} , that is $D^b(A)$ is triangle-equivalent to the derived category $D^b(\mathcal{H})$ of bounded complexes over \mathcal{H} . Over the years, piecewise hereditary algebras have been widely investigated and proved to be related with many other topics, such as the simply connected algebras and the trivial extensions, the self-injective algebras of polynomial growth and the strong global dimension.

Hereditary categories \mathcal{H} having tilting objects are of special interest in representation theory of algebras. The endomorphism algebras $\text{End}_{\mathcal{H}} T$ of tilting objects T in \mathcal{H} , called *quasitilted* algebras, were introduced and studied in [8]. It is well-known that \mathcal{H} and $\text{End}_{\mathcal{H}} T$ are derived equivalent. When k is algebraically closed, it was shown by Happel [6] that \mathcal{H} is either derived equivalent to a finite dimensional hereditary k -algebra H or derived equivalent to a category of coherent sheaves $\text{coh } \mathbb{X}$ on a weighted projective line \mathbb{X} (in the sense of [3]).

The aim of this paper is to study the skew group algebra $A[G]$ (see Section 1.1), in case A is a piecewise hereditary algebra. Our main result (Theorem 2) shows that under standard assumptions, the skew group algebra $A[G]$ is also piecewise hereditary.

In order to give a clear statement of our main results, we need additional terminology. Let G be a group and \mathcal{A} be an additive category. An *action* of G on \mathcal{A} is a group homomorphism $\theta : G \rightarrow \text{Aut } \mathcal{A}$ ($\sigma \mapsto \theta_\sigma$) from G to the group of automorphisms of \mathcal{A} . An object \mathcal{M} in \mathcal{A} is *G -stable* with respect to θ , or briefly *G -stable* in case there is no ambiguity, if $\theta_\sigma \mathcal{M} \cong \mathcal{M}$ for all $\sigma \in G$. For such an object, the algebra $B = \text{End}_{\mathcal{A}} \mathcal{M}$ inherits an action of G from θ , denoted by θ_B ; see 1.1.2. Also, given another additive category \mathcal{B} and an action $\vartheta : G \rightarrow \text{Aut } \mathcal{B}$ ($\sigma \mapsto \vartheta_\sigma$) of G on \mathcal{B} , a functor $F : \mathcal{A} \rightarrow \mathcal{B}$ is *G -compatible*, with respect to the pair (θ, ϑ) , if $F\theta_\sigma = \vartheta_\sigma F$ for every $\sigma \in G$.

Examples of particular interest occur when a group G acts on an artin algebra A as above. Then the action θ of G on A induces an action $\theta_{\text{mod } A}$ of G on $\text{mod } A$, and further an action $\theta_{D^b(A)}$ on $D^b(A)$; see Sections 1.1 and 2.1.

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This opens the way to our main results. Our first theorem stands as a generalization of the main result in [9,7].

Theorem 1. Let A be a k -algebra, and $\mathcal{H} = \text{mod } H$, with H a hereditary algebra, or $\mathcal{H} = \text{coh } \mathbb{X}$, the category of coherent sheaves on a weighted projective line \mathbb{X} . Let G be a group, and $\theta : G \rightarrow \text{Aut } A$ and $\vartheta : G \rightarrow \text{Aut } \mathcal{H}$ be fixed actions of G on A and \mathcal{H} .

(a) The following conditions are equivalent:

- (i) There exists a G -compatible triangle-equivalence $E : D^b(\mathcal{H}) \rightarrow D^b(A)$ (with respect to the pair of induced actions $(\vartheta_{D^b(\mathcal{H})}, \theta_{D^b(A)})$);
 - (ii) There exist a G -stable tilting object T in \mathcal{H} and sequences $\text{End}_{\mathcal{H}} T = A_0, A_1, \dots, A_n = A$ of k -algebras and T_0, T_1, \dots, T_{n-1} of modules such that, for each i , $A_{i+1} = \text{End}_{A_i} T_i$ and T_i is a G -stable tilting or cotilting A_i -module (with respect to the induced action $\vartheta_{\text{mod } A_i}$), and the induced action ϑ_{A_n} coincides with θ ;
 - (iii) There exist a G -stable tilting object T in \mathcal{H} and sequences $\text{End}_{\mathcal{H}} T = A_0, A_1, \dots, A_n = A$ of k -algebras and T_0, T_1, \dots, T_{n-1} of modules such that, for each i , $A_{i+1} = \text{End}_{A_i} T_i$ and T_i is a G -stable splitting tilting or cotilting A_i -module (with respect to the induced action $\vartheta_{\text{mod } A_i}$), and the induced action ϑ_{A_n} coincides with θ ;
- (b) If the conditions in (a) are satisfied and G is a finite group whose order is not a multiple of the characteristic of k , then the algebra $(\text{End}_{\mathcal{H}} T)[G]$, where T is as in (ii) or in (iii), is quasitilted and derived equivalent to $A[G]$. In particular, $A[G]$ is piecewise hereditary.

The equivalence of the conditions (a)(i)–(a)(iii) of **Theorem 1** was previously shown in [7,9] in the case where, essentially, the actions θ and ϑ are the trivial actions, that is trivial homomorphisms of groups. Actually, our proofs are adaptations of the original ones.

In addition, it will become clear in Section 4 that any triangle-equivalence $D^b(\mathcal{H}) \rightarrow D^b(A)$ can be converted into a G -compatible triangle-equivalence. As an application of this observation, together with **Theorem 1** and Happel's Theorem [6], we will obtain our main theorem.

Theorem 2. Let A be a piecewise hereditary k -algebra of type \mathcal{H} , for some Ext-finite hereditary abelian k -category with tilting objects \mathcal{H} . Moreover, let G be a finite group whose order is not a multiple of the characteristic of k . Then,

- (a) If $\mathcal{H} = \text{mod } H$, for some hereditary algebra H , then for any action of G on A , there exist a hereditary algebra H' , derived equivalent to H , and an action of G on H' such that $A[G]$ is piecewise hereditary of type $\text{mod } H'[G]$.
- (b) If $\mathcal{H} = \text{coh } \mathbb{X}$, for some category of coherent sheaves on a weighted projective line \mathbb{X} , then for any action of G on A there exist an action of G on \mathcal{H} and a G -compatible triangle-equivalence $E : D^b(\mathcal{H}) \rightarrow D^b(A)$. In particular, $A[G]$ is piecewise hereditary.

In Section 1, we fix the notations and terminologies. Most of the necessary background on weighted projective lines is however postponed to Section 4, since it is not explicitly needed until then. In Section 2, we study the G -compatible triangle-equivalences of derived categories induced by the G -stable tilting modules. Section 3 is devoted to the proof of **Theorem 1**. In Section 4, we prove **Theorem 2**. This involves showing that any triangle-equivalence between $D^b(\mathcal{H})$ and $D^b(A)$ can be converted into a G -compatible equivalence when \mathcal{H} is a module category over a hereditary algebra or a category of coherent sheaves on a weighted projective line. Finally, in Section 4.2, we give an illustrative example.

1. Preliminaries

In this paper, all considered algebras are finite dimensional algebras over an algebraically closed field k (and, unless otherwise specified, basic and connected). Moreover, all modules are finitely generated left modules. For an algebra A , we denote by $\text{proj } A$ a full subcategory of $\text{mod } A$ consisting of one representative from each isomorphism class of indecomposable projective modules. Given an A -module T , we let $\text{add } T$ be the full subcategory of $\text{mod } A$ having as objects the direct sums of indecomposable direct summands of T . Also, the functor $D = \text{Hom}_k(-, k)$ is the standard duality between $\text{mod } A$ and $\text{mod } A^{\text{op}}$.

Let A be an algebra. An A -module T is a *tilting module* if T has projective dimension at most one, $\text{Ext}_A^1(T, T) = 0$ and there exists a short exact sequence of A -modules $0 \rightarrow A \rightarrow T_0 \rightarrow T_1 \rightarrow 0$ in $\text{mod } A$, with $T_0, T_1 \in \text{add } T$.

For basic results on tilting theory, we refer to [2], and on derived categories we refer to [5] or [15]. For an object M in a triangulated category, we shall denote the image of M under the “shift” self-equivalence T by $M[1]$, and similarly $T^n M$ will be denoted by $M[n]$ for any n .

1.1. Skew group algebras

Let A be an algebra and G be a group with identity σ_1 . We consider an *action* of G on A , that is a function $G \times A \rightarrow A$, $(\sigma, a) \mapsto \sigma(a)$, such that:

- (a) For each σ in G , the map $\sigma : A \rightarrow A$ is an automorphism of algebra;
- (b) $(\sigma\sigma')(a) = \sigma(\sigma'(a))$ for all $\sigma, \sigma' \in G$ and $a \in A$;
- (c) $\sigma_1(a) = a$ for all $a \in A$.

For any such action, the *skew group algebra* $A[G]$ is the free left A -module with basis all the elements in G endowed with the multiplication given by $(a\sigma)(b\sigma') = a\sigma(b)\sigma\sigma'$ for all $a, b \in A$ and $\sigma, \sigma' \in G$. Clearly, $A[G]$ admits a structure of right A -module. Observe that $A[G]$ is generally not connected and basic, but this will not play any major role in the sequel.

In addition, any action of G on A induces a group action on $\text{mod } A$: for any $M \in \text{mod } A$ and $\sigma \in G$, let ${}^\sigma M$ be the A -module with the additive structure of M and with the multiplication $a \cdot m = \sigma^{-1}(a)m$, for $a \in A$ and $m \in M$. Given a morphism of A -modules $f : M \rightarrow N$, define ${}^\sigma f : {}^\sigma M \rightarrow {}^\sigma N$ by ${}^\sigma f(m) = f(m)$ for each $m \in {}^\sigma M$. This defines an action of G on $\text{mod } A$; see [1].

When $G = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$ is a finite group, the natural inclusion of A in $A[G]$ induces the change of ring functors $F = A[G] \otimes_A - : \text{mod } A \rightarrow \text{mod } A[G]$ and $H = \text{Hom}_{A[G]}(A[G], -) : \text{mod } A[G] \rightarrow \text{mod } A$. These have been extensively studied in [1,11,14], for instance, assuming the order of G is not a multiple of the characteristic of k . We recall the following facts from [11, (1.1)(1.8)].

Remark 1.1.1. (a) (F, H) and (H, F) are adjoint pairs of functors.

(b) Let $M \in \text{mod } A$ and $\sigma \in G$. The subset $\sigma \otimes_A M = \{\sigma \otimes_A m \mid m \in M\}$ of FM has a structure of A -module given by $a(\sigma \otimes_A m) = \sigma\sigma^{-1}(a) \otimes_A m = \sigma \otimes_A \sigma^{-1}(a)m = \sigma \otimes_A (a \cdot m)$, so that $\sigma \otimes_A M$ and ${}^\sigma M$ are isomorphic as A -modules. Therefore, as A -modules, we have

$$FM \cong \bigoplus_{i=1}^n (\sigma_i \otimes_A M) \cong \bigoplus_{i=1}^n {}^{\sigma_i} M.$$

Then, $HFM \cong \bigoplus_{i=1}^n (\sigma_i \otimes_A M) \cong \bigoplus_{i=1}^n {}^{\sigma_i} M$.

(c) Given a morphism $f : M \rightarrow N$ and $\sigma \in G$, the map ${}^\sigma f : {}^\sigma M \rightarrow {}^\sigma N$ becomes $\sigma \otimes M \rightarrow \sigma \otimes N : \sigma \otimes m \mapsto \sigma \otimes f(m)$ (that we also denote ${}^\sigma f$) under the isomorphism ${}^\sigma M \cong \sigma \otimes M$. In the following, we will freely use both notations.

We also need the following key observation.

Remark 1.1.2. In what follows, it will be convenient to consider the case where a group G acts on a Krull–Schmidt category \mathcal{A} (with, say, ${}^\sigma(-) : \mathcal{A} \rightarrow \mathcal{A}$ for $\sigma \in G$) and T is a (basic) G -stable object in \mathcal{A} , that is, for each $\sigma \in G$, we have an isomorphism $\alpha_\sigma : T \rightarrow {}^\sigma T$. In this case, $\text{End}_{\mathcal{A}} T$ is naturally endowed with an action of G , given by $\sigma(f) = \alpha_\sigma^{-1} \circ {}^\sigma f \circ \alpha_\sigma$, for $f \in \text{End}_{\mathcal{A}} T$.

2. Group actions and G -compatible derived equivalences

In this section, we recall how an action of G on an additive category \mathcal{A} induces an action of G on the homotopy and derived categories of \mathcal{A} . Once this is done, we show that the equivalences of derived categories induced by G -stable tilting modules are G -compatible.

2.1. Group actions on homotopy and derived categories

Let G be a group and assume that \mathcal{A} is an additive category on which G acts. For each $\sigma \in G$, let ${}^\sigma(-) : \mathcal{A} \rightarrow \mathcal{A}$ be the automorphism of \mathcal{A} induced by σ . For any complex $M^\bullet = (M^i, d_{M^\bullet}^i)_{i \in \mathbb{Z}}$ over \mathcal{A} and $\sigma \in G$, let ${}^\sigma M^\bullet$ be the complex $({}^\sigma M^i, {}^\sigma d_{M^\bullet}^i)_{i \in \mathbb{Z}}$. Moreover, given another complex $N^\bullet = (N^i, d_{N^\bullet}^i)_{i \in \mathbb{Z}}$ and a morphism of complexes $f = (f^i : M^i \rightarrow N^i)_{i \in \mathbb{Z}}$, let ${}^\sigma f = ({}^\sigma f^i : {}^\sigma M^i \rightarrow {}^\sigma N^i)_{i \in \mathbb{Z}}$. Clearly, ${}^\sigma f$ is a morphism of complexes. Since ${}^\sigma(-) : \mathcal{A} \rightarrow \mathcal{A}$ is an automorphism, this construction is compatible with the homotopy relation. This allows defining, for each $\sigma \in G$, an endomorphism ${}^\sigma(-) : K^b(\mathcal{A}) \rightarrow K^b(\mathcal{A})$. Moreover, since this action preserves the quasi-isomorphisms, it extends to an action on $D^b(\mathcal{A})$.

Proposition 2.1.1. Let $\sigma \in G$. The mapping $M^\bullet \mapsto {}^\sigma M^\bullet$ (where M^\bullet is a complex over \mathcal{A}) induces an action of G on $D^b(\mathcal{A})$. In addition, the automorphisms ${}^\sigma(-) : D^b(\mathcal{A}) \rightarrow D^b(\mathcal{A})$ induced by the elements $\sigma \in G$ are triangle-equivalences.

At this point, recall that if $\mathcal{A} = \text{mod } A$, for some finite dimensional k -algebra A of finite global dimension (for instance if A is piecewise hereditary [7, (1.2)]), then $D^b(\mathcal{A})$ has almost split triangles. We have the following result.

Proposition 2.1.2. Let \mathcal{A} be as above and $\sigma \in G$. Then, the automorphism ${}^\sigma(-) : D^b(\mathcal{A}) \rightarrow D^b(\mathcal{A})$ preserves the almost split triangles.

We get the following corollary, where the proof follows directly from 2.1.2.

Corollary 2.1.3. Let \mathcal{A} be as above and $\sigma \in G$. Then the Auslander–Reiten translation τ and the functor ${}^\sigma(-)$ commute on objects. In particular, the functor ${}^\sigma(-)$ preserves the τ -orbits in the Auslander–Reiten quiver of $D^b(\mathcal{A})$.

2.2. G -compatible derived equivalences

It is well-known from [5] that any tilting module induces an equivalence of derived categories. Here, we show that the G -stable tilting modules induce G -compatible equivalences. We recall the following facts from [5, (III.2)]: let A be a finite

dimensional k -algebra of finite global dimension. Given a tilting A -module T and $B = \text{End}_A T$, the functors

- (i) $\text{Hom}_A(T, -) : K^b(\text{add} T) \longrightarrow K^b(\text{proj} B)$
- (ii) $\rho : K^b(\text{proj} B) \hookrightarrow K^b(\text{mod } B) \longrightarrow D^b(B)$
- (iii) $\phi : K^b(\text{add} T) \hookrightarrow K^b(\text{mod } A) \longrightarrow D^b(A)$

are equivalences of triangulated categories, and the composition

- (iv) $\text{RHom}(T, -) = \rho \circ \text{Hom}_A(T, -) \circ \phi^{-1} : D^b(A) \longrightarrow D^b(B)$ takes T to B .

Observe that above, and below, the functor $\text{Hom}_A(T, -)$ is the component-wise functor taking a complex $T^\bullet = (T^i, f^i)$ in $K^b(\text{add} T)$ to the complex $\text{Hom}_A(T, T^\bullet) = (\text{Hom}_A(T, T^i), \text{Hom}_A(T, f^i))$ in $K^b(\text{proj} B)$.

Proposition 2.2.1. *Let A be an algebra and G be a group acting on A . If T is a tilting A -module which is G -stable with respect to the induced action of G on $\text{mod } A$, then the equivalences (i)–(iv) given above are G -compatible.*

Proof. First, let $\theta : G \longrightarrow \text{Aut } A$ be an action of G on A and, for each $\sigma \in G$, let ${}^\sigma(-) : \text{mod } A \longrightarrow \text{mod } A$ be the induced automorphism. Moreover, let T be a tilting A -module which is G -stable with respect to this action, endowed with isomorphisms $\alpha_\sigma : T \xrightarrow{\sim} \sigma \otimes T$ for $\sigma \in G$ as in 1.1.2. The G -stability of T gives rise to a natural action of G on $\text{add } T$. Then, following Section 2.1, the additive category $K^b(\text{add } T)$ inherits a (component-wise) action of G . We also denote the induced automorphism on $K^b(\text{add } T)$ by ${}^\sigma(-)$, for $\sigma \in G$. Also, since T is G -stable, it follows from 1.1.2 that $B = \text{End}_A T$ is endowed with a natural action of G , which we extend to $K^b(\text{proj } B)$ and $D^b(B)$. Again, we denote the induced automorphisms by ${}^\sigma(-)$, for $\sigma \in G$.

(i) $\text{Hom}_A(T, -) : K^b(\text{add} T) \longrightarrow K^b(\text{proj} B)$. Let $\sigma \in G$ and $T^\bullet = (T^i, f^i)$ be a complex in $K^b(\text{add} T)$. We need to verify that $\sigma \otimes_B \text{Hom}_A(T, T^\bullet) \cong \text{Hom}_A(T, \sigma \otimes T^\bullet)$ functorially. To do so, for each i consider the map β^i from $\sigma \otimes_B \text{Hom}_A(T, T^i)$ to $\text{Hom}_A(\sigma \otimes_A T, \sigma \otimes_A T^i)$ taking $\sigma \otimes g$ onto ${}^\sigma g$ where ${}^\sigma g$ is as in 1.1.1(c). Then β^i is a morphism of B -modules. Since β^i is clearly bijective, it is an isomorphism of B -modules. Since the β^i 's commute with any morphism in $\text{add } T$, we have a functorial isomorphism of complexes β^\bullet . Now, since T is G -stable, we have $T \cong \sigma \otimes T$ and a functorial isomorphism

$$\gamma^\bullet : \sigma \otimes_B \text{Hom}_A(T, T^\bullet) \xrightarrow{\beta^\bullet} \text{Hom}_A(\sigma \otimes T, \sigma \otimes T^\bullet) \xrightarrow{\sim} \text{Hom}_A(T, \sigma \otimes T^\bullet),$$

showing that $\text{Hom}_A(T, -)$ is G -compatible.

(ii) and (iii). Since the inclusions and localization functors are clearly G -compatible, so are ρ and ϕ . \square

3. Piecewise hereditary algebras revisited

The aim of this section is to prove Theorem 1. Before doing so, we need to recall some facts concerning skew group algebras and prove preliminary results.

3.1. Preliminary results

Let A be an algebra and $G = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$ be a finite group acting on A whose order is not a multiple of the characteristic of k . Let σ_1 be the unit of G , T be an A -module and $f : FT \longrightarrow FT$ be a k -linear morphism. Since $FT \cong \bigoplus_{i=1}^n (\sigma_i \otimes_A T)$, f is given by a matrix $f = (f_{\sigma_i, \sigma_j})_{1 \leq i, j \leq n}$ where each f_{σ_i, σ_j} is a morphism from $\sigma_j \otimes T$ to $\sigma_i \otimes T$. Clearly, f is A -linear if and only if each f_{σ_i, σ_j} is A -linear. Now, if f is A -linear, then it is $A[G]$ -linear if and only if $f(\sigma_k \sigma_j \otimes t) = \sigma_k f(\sigma_j \otimes t)$ for all $\sigma_j, \sigma_k \in G$, and quick computations show that this is the case if and only if $f_{\sigma_k \sigma_i, \sigma_k \sigma_j}(\sigma_k \sigma_j \otimes t) = \sigma_k \cdot f_{\sigma_i, \sigma_j}(\sigma_j \otimes t)$ for all $\sigma_i, \sigma_j, \sigma_k \in G$ and $t \in T$. In particular, f is determined by $\{f_{\sigma_1, \sigma_1}, \dots, f_{\sigma_1, \sigma_n}\}$.

Proposition 3.1.1. *Let A and G be as above, and T be a G -stable (basic) tilting A -module with isomorphisms $\alpha_\sigma : T \xrightarrow{\sim} \sigma \otimes T$ as in 1.1.2. Then,*

- (a) FT is a $A[G]$ -tilting module.
- (b) $\text{End}_{A[G]} FT \cong (\text{End}_A T)[G]$.

Proof. (a) Since F is exact and preserves the projectives by 1.1.1, FT has projective dimension at most one. In addition, we have

$$\text{Ext}_{A[G]}^1(FT, FT) \cong D\text{Hom}_{A[G]}(FT, \tau(FT)) \cong D\text{Hom}_{A[G]}(FT, F(\tau T)),$$

by the Auslander–Reiten formula and [11, (4.2)]. By adjunction, this latter group is nonzero if and only if $\text{Hom}_A(T, HF(\tau T)) \cong \text{Hom}_A(T, \bigoplus_{\sigma \in G} {}^\sigma(\tau T))$ is nonzero, where the isomorphism follows from 1.1.1. However, if $f : T \xrightarrow{\sim} {}^\sigma(\tau T)$ is a nonzero morphism, with $\sigma \in G$, then $\sigma^{-1} f \circ \alpha_{\sigma^{-1}} : T \xrightarrow{\sim} \tau T$ is nonzero, a contradiction to $\text{Ext}_A^1(T, T) = 0$. So, $\text{Ext}_{A[G]}^1(FT, FT) = 0$. Finally, any short exact sequence $0 \longrightarrow A \longrightarrow T_0 \longrightarrow T_1 \longrightarrow 0$ in $\text{mod } A$, with $T_0, T_1 \in \text{add } T$ induces a short exact sequence $0 \longrightarrow A[G] \longrightarrow FT_0 \longrightarrow FT_1 \longrightarrow 0$ in $\text{mod } A[G]$. So FT is a tilting $A[G]$ -module.

(b) In order to show that $\text{End}_{A[G]}FT$ and $(\text{End}_A T)[G]$ are isomorphic algebras, we construct explicit inverse isomorphisms between them. Let $f \in \text{End}_{A[G]}FT$, and assume that f is given by a matrix $f = (f_{\sigma_i, \sigma_j})_{1 \leq i, j \leq n}$, where each f_{σ_i, σ_j} is a morphism from $\sigma_j \otimes T$ to $\sigma_i \otimes T$. For each i , let $f_i = f_{\sigma_1, \sigma_i} \circ \alpha_{\sigma_i} : T \rightarrow \sigma_1 \otimes T = T$, and define $\nu : \text{End}_{A[G]}FT \rightarrow (\text{End}_A T)[G]$ by $\nu(f) = \sum_{i=1}^n f_i \sigma_i$ for each $f \in \text{End}_{A[G]}FT$.

Conversely, let $\sum_{i=1}^n f_i \sigma_i \in (\text{End}_A T)[G]$, and consider the family of morphisms $\{f_{\sigma_1, \sigma_1}, \dots, f_{\sigma_1, \sigma_n}\}$, where $f_{\sigma_1, \sigma_i} := f_i \circ \alpha_{\sigma_i}^{-1}$ for each i . These morphisms determine an $A[G]$ -linear map f from FT to FT . Hence, define $\mu : (\text{End}_A T)[G] \rightarrow \text{End}_{A[G]}FT$ by $\mu(\sum_{i=1}^n f_i \sigma_i) = f$ for each $\sum_{i=1}^n f_i \sigma_i \in (\text{End}_A T)[G]$.

Clearly, ν and μ are inverse constructions, preserving sums and units. It remains to show that ν preserves the product. For $f = (f_{\sigma_i, \sigma_j})_{1 \leq i, j \leq n}$ and $g = (g_{\sigma_i, \sigma_j})_{1 \leq i, j \leq n}$, let $\nu(f) = \sum_{i=1}^n f_i \sigma_i$ and $\nu(g) = \sum_{j=1}^n g_j \sigma_j$. Then,

$$\begin{aligned} \nu(f) \cdot \nu(g) &= \sum_{i=1}^n \sum_{j=1}^n f_i \sigma_i (g_j \sigma_j) \sigma_i \sigma_j \\ &= \sum_{i=1}^n \sum_{j=1}^n f_i \circ (\alpha_{\sigma_i}^{-1} \circ \alpha_{\sigma_j} \circ g_j \circ \alpha_{\sigma_i}) \sigma_i \sigma_j \\ &= \sum_{i=1}^n \sum_{j=1}^n (f_{\sigma_1, \sigma_i} \circ \alpha_{\sigma_i}^{-1} \circ g_{\sigma_1, \sigma_j} \circ \alpha_{\sigma_j} \circ \alpha_{\sigma_i}) \sigma_i \sigma_j \\ &= \sum_{k=1}^n \left(\sum_{i=1}^n (f_{\sigma_1, \sigma_i} \circ g_{\sigma_1, \sigma_k} \circ \alpha_{\sigma_k}) \right) \sigma_k \\ &= \nu(f \cdot g). \quad \square \end{aligned}$$

3.2. Proof of Theorem 1

Let \mathcal{A} be an arbitrary abelian category, and $M \in \mathcal{A}$ satisfying $\text{Ext}^1(M, M) = 0$ and $\text{Ext}^2(M, N) = 0$ for all $N \in \mathcal{A}$. Then, the right perpendicular category M^\perp is the full subcategory of \mathcal{A} containing the objects N satisfying $\text{Hom}(M, N) = \text{Ext}^1(M, N) = 0$. We define dually the left perpendicular category ${}^\perp M$. It was shown in [4] that M^\perp and ${}^\perp M$ are again abelian categories.

We can now proceed with the proof. The cases $\mathcal{H} = \text{mod } H$ and $\mathcal{H} = \text{coh } \mathbb{X}$ are treated separately. Observe that our proof is an adaptation of the proofs of the main results in [9,7], respectively, and to which we freely refer in the course of the proof.

Proof of Theorem 1. (a) Clearly, (iii) implies (ii), while (ii) implies (i) by an easy induction using 2.2.1 and its dual.

Now, let $\mathcal{H} = \text{mod } H$, for some hereditary algebra H , and suppose that (i) holds. Assume, without loss of generality, that A and H are basic and connected. Let $E : D^b(H) \rightarrow D^b(A)$ be a G -compatible equivalence; we shall identify the module categories $\text{mod } A$ and $\text{mod } H$ with their images under the natural embeddings into $D^b(A)$ and $D^b(H)$, respectively.

Let M^\bullet be an object of $D^b(H)$ such that EM^\bullet is isomorphic to A and assume, without loss of generality, that $M^\bullet = M_0 \oplus M_1[1] \oplus \dots \oplus M_r[r]$ where the M_i are H -modules and $M_0 \neq 0 \neq M_r$. Note that $\text{Hom}_H(M_i, M_j) = 0$ if $i \neq j$, and $\text{Ext}_H^1(M_i, M_j) = 0$ if $i+1 \neq j$. Also, since A is G -stable, then so is M^\bullet and thus, since the $M_i[i]$ lie in different degrees, each of them is also G -stable.

We prove our claim by induction on r . If $r = 0$, then one can check that $M^\bullet = M_0$ is a G -stable tilting H -module, and so $A \cong \text{End}_H M_0$. In addition, since the isomorphism is given by E , the actions of G on A and $\text{End}_H M_0$ coincide. Also, since H is hereditary, then M^\bullet is splitting and the result follows.

Now, assume inductively that the result holds true whenever r takes a smaller value, or r takes the same value and M_0 has less indecomposable direct summands. We shall find it convenient to construct a sequence of separating tilting modules instead of splitting tilting modules. We recall that T_i is a separating tilting A_i -module with endomorphism ring A_{i+i} if and only if T_i is a splitting tilting A_{i+i}^{op} -module with endomorphism ring A_i^{op} .

Let $L = M_1 \oplus M_2 \oplus \dots \oplus M_r$. Then, by [9, (Proposition 3)], the subcategory L^\perp of $\text{mod } H$ is equivalent to $\text{mod } A$, for some finite dimensional hereditary algebra A . Moreover, the inclusion functor $\text{mod } A \hookrightarrow \text{mod } H$ is full, faithful and exact. In what follows, we identify L^\perp and $\text{mod } A$. Observe that $M_0 \in \text{mod } A$, and in fact is a tilting A -module. Also, $\text{mod } A$ is G -stable since so is L .

Let $Q = \text{Hom}_k(A, k)$, a minimal injective cogenerator for $\text{mod } A$, and let

$$T^\bullet = Q \oplus M_1[1] \oplus M_2[2] \oplus \dots \oplus M_r[r].$$

We observe that Q is a G -stable A -module. Indeed, it is easily verified that if I is an injective A -module, then so is ${}^\sigma I$ for each $\sigma \in G$. Because each M_i is also G -stable, then so is T^\bullet . Following [9], ET^\bullet is (isomorphic to) a separating tilting A -module. Moreover, ET^\bullet is G -stable since E is G -compatible and T^\bullet is G -stable. Now let $B = \text{End}_A ET^\bullet$. By 2.2.1, we have an equivalence

$$\begin{array}{ccccc} E'' : & D^b(H) & \xrightarrow{E} & D^b(A) & \xrightarrow{E'} & D^b(B) \\ & T^\bullet & \mapsto & ET^\bullet & \mapsto & B \end{array}$$

which is G -compatible since so are E and E' . Let Q_0 be a simple indecomposable direct summand of Q : such a module exists since Λ is hereditary. Then, for each $\sigma \in G$, the Λ -module ${}^\sigma Q_0$ is also simple injective and $E''({}^\sigma Q_0) = {}^\sigma E''(Q_0)$ is a simple projective B -module. Since B is connected, $E''({}^\sigma Q_0)$ is not an injective B -module unless we are in the trivial case of a simple algebra. Observe moreover that since each ${}^\sigma Q_0$ is a simple injective Λ -module, then the set $\{{}^\sigma Q_0 \mid \sigma \in G\}$ is finite, and we denote its cardinality by n .

Now, imitating the arguments of [9], we find, for each $\sigma \in G$, an object R_σ in $D^b(H)$ isomorphic to $U_\sigma[1]$, for some H -module U_σ , and such that

$$E''R_\sigma \cong \tau^{-1}E''({}^\sigma Q_0) \cong \tau^{-1}({}^\sigma(E''Q_0)) \cong {}^\sigma(\tau^{-1}E''Q_0).$$

So $\bigoplus_{\sigma \in G} E''R_\sigma$ is a G -stable B -module. We let $S = (\bigoplus_{\sigma \in G} E''R_\sigma) \oplus E''N$, where $(\bigoplus_{\sigma \in G} {}^\sigma Q_0) \oplus N = T^\bullet$.

At this point, it is worthwhile to observe that $(\bigoplus_{\sigma \in G} R_\sigma) \oplus N = N_0 \oplus N_1[1] \oplus M_2[2] \oplus \cdots \oplus M_r[r]$ for some $N_0, N_1 \in \text{mod } H$, and where, by definition of the R_σ , N_0 has n less indecomposable direct summands than M_0 . In addition, by construction, S is a “generalized” APR-tilting B -module; what is important for our purpose, and easy to verify, is that S is a separating tilting B -module.

Let $C = \text{End}_B(S)$. By 2.2.1, we have an equivalence of triangulated categories

$$E''' : \quad D^b(H) \xrightarrow{E''} D^b(B) \xrightarrow{\cong} D^b(C) \\ (\bigoplus_{\sigma \in G} R_\sigma) \oplus N \mapsto S \mapsto C$$

which is G -compatible. Moreover, as observed earlier, we have $(\bigoplus_{\sigma \in G} R_\sigma) \oplus N = N_0 \oplus N_1[1] \oplus M_2[2] \oplus \cdots \oplus M_r[r]$ where $N_0 = 0$ or contains less indecomposable direct summands than M_0 . By induction hypothesis, C is piecewise hereditary of type $\text{mod } H$ and, using the separating tilting modules ET^\bullet and S , and keeping in mind our preceding discussion on separating tilting modules, so is A . This shows the equivalence of conditions (i), (ii) and (iii), when $\mathcal{H} = \text{mod } H$.

We now assume that $\mathcal{H} = \text{coh } \mathbb{X}$, for some weighted projective line \mathbb{X} , and that condition (i) holds. We will give all details until the case $\text{mod } H$ carries over.

Let $E : D^b(\mathcal{H}) \rightarrow D^b(A)$ be a G -compatible equivalence of triangulated categories. Let M^\bullet be an object of $D^b(\mathcal{H})$ such that EM^\bullet is isomorphic to A and assume, without loss of generality, that $M^\bullet = M_0 \oplus M_1[1] \oplus \cdots \oplus M_r[r]$ for some $M_i \in \mathcal{H}$, with $M_0 \neq 0 \neq M_r$. Note that $\text{Hom}_{\mathcal{H}}(M_i, M_j) = 0$ if $i \neq j$, and $\text{Ext}_{\mathcal{H}}^1(M_i, M_j) = 0$ if $i + 1 \neq j$. Also, since A is G -stable, so is M^\bullet , and thus, since the $M_i[i]$ lie in different degrees, each of them is also G -stable.

We prove our claim by induction on r . If $r = 0$, then one can check that $M^\bullet = M_0$ is a G -stable tilting object in \mathcal{H} , and $A \cong \text{End}_{\mathcal{H}} M_0$. So A is quasitilted. In addition, since the isomorphism is given by E , the actions of G on A and $\text{End}_{\mathcal{H}} M_0$ coincide.

Assume inductively that the result holds true in all cases where either r takes a smaller value, or r takes the same value and either M_0 or M_r has less indecomposable direct summands. Then, M_0 is a G -stable tilting object in the abelian category $(\bigoplus_{i=1}^r M_i[i])^\perp$ and M_r is a G -stable cotilting object in the abelian category ${}^\perp(\bigoplus_{i=0}^{r-1} M_i[i])$. By [7], one of these categories is a module category over a hereditary artin algebra H . The situation is then reduced to the case $\mathcal{H} = \text{mod } H$, and we are done. This proves (a).

(b) Now, assume that the equivalent conditions of (a) are satisfied and that G is a finite group whose order is not a multiple of the characteristic of k . Then there exists a tilting object T in \mathcal{H} and a sequence of algebras $\text{End}_{\mathcal{H}} T = A_0, A_1, \dots, A_n = A$ on which G acts and a sequence T_0, T_1, \dots, T_{n-1} where T_i is a G -stable tilting or cotilting A_i -module with endomorphism ring isomorphic to A_{i+1} for each i . Moreover, by 3.1.1 and its dual, $A_i[G] \otimes_{A_i} T_i$ is a tilting or cotilting $A_i[G]$ -module for each i and $\text{End}_{A_i[G]}(A_i[G] \otimes_{A_i} T_i) \cong A_{i+1}[G]$. Now, since the order of G is not a multiple of the characteristic of k , $A_0[G]$ is hereditary if $\mathcal{H} = \text{mod } H$ by [11, (1.3)], and quasitilted if $\mathcal{H} = \text{coh } \mathbb{X}$ by [8, (III.1.6)]. The statement thus follows from [9] and [7], respectively. \square

4. Main result

4.1. Proof of Theorem 2

Assume that the hypotheses of Theorem 2 are satisfied. In view of Theorem 1, it would be sufficient to prove Theorem 2 to show that there is a G -compatible equivalence between $D^b(A)$ and $D^b(\mathcal{H})$. In this section, we show that this holds for $\mathcal{H} = \text{coh } \mathbb{X}$, and show that when $\mathcal{H} = \text{mod } H$, it is however possible to construct a derived equivalent hereditary algebra H' on which G acts and for which there is a G -compatible equivalence between $D^b(A)$ and $D^b(H')$.

The first situation we consider is that of a piecewise hereditary algebra A of type $\mathcal{H} = \text{mod } H$, for some hereditary algebra H . Here, it will be sufficient to assume that G is a torsion group acting on A . Let $Q = (Q_0, Q_1)$ be a finite and acyclic quiver such that $H \cong kQ$, where Q_0 and Q_1 respectively denote the set of vertices and arrows of Q . All directed components are isomorphic to $\mathbb{Z}Q$ as translation quivers, and so have only finitely many τ -orbits. Our first aim is to show that any such directed component Γ admits a section which is stable under the induced action of G on $D^b(A)$. Recall that a full and connected subquiver Ω of Γ is a *section* if it contains no oriented cycles, it intersects each τ -orbit of Γ exactly once and it is convex.

By the above, we remark that, if we set ${}^\sigma \Gamma := \{{}^\sigma M^\bullet \mid M^\bullet \in \Gamma\}$, then ${}^\sigma \Gamma = \Gamma$ for every $\sigma \in G$. We now construct a G -stable section in Γ as follows.

Definition 4.1.1. Let G, A and Γ be as above, and let X^\bullet be a fixed object in Γ . We define $\Sigma (= \Sigma_{X^\bullet})$ to be the full subquiver of Γ formed by the objects M^\bullet in Γ such that there exists a path ${}^\sigma X^\bullet \rightsquigarrow M^\bullet$ for some $\sigma \in G$ and any such path is sectional.

Lemma 4.1.2. Let Σ and Γ be as above. Then Σ intersects each τ -orbit of Γ exactly once.

Proof. Let $M^\bullet \in \Gamma$. Since Γ is directed, for each $\sigma \in G$, there exists an integer r_σ such that there exists a path from ${}^\sigma X^\bullet$ to $\tau^{r_\sigma} M^\bullet$ in Γ if and only if $r \leq r_\sigma$. Clearly, any path from ${}^\sigma X^\bullet$ to $\tau^{r_\sigma} M^\bullet$ is sectional. There exists an integer s which is maximal for the property that there exists a path ${}^\sigma X^\bullet \rightsquigarrow \tau^s M^\bullet$ in Γ , for some $\sigma \in G$. The maximality of s gives $\tau^s M^\bullet \in \Sigma$. The uniqueness of $\tau^s M^\bullet$ follows from the definition of Σ . \square

Lemma 4.1.3. Let Σ and Γ be as above. Let $\omega : M^\bullet \longrightarrow M_1^\bullet \longrightarrow \dots \longrightarrow M_n^\bullet$ be a walk in Γ , with $n \geq 1$ and $M^\bullet \in \Sigma$. Then $\tau^k M_n^\bullet \in \Sigma$ for some integer k . Moreover, M^\bullet and $\tau^k M_n^\bullet$ belong to the same connected component of Σ .

Proof. Let ω be as in the statement. We prove our claim by induction. First observe that by Lemma 4.1.2, it follows that if $f : M^\bullet \longrightarrow N^\bullet$ is an irreducible morphism in Γ , with $M^\bullet \in \Sigma$, then $N^\bullet \in \Sigma$ or $\tau N^\bullet \in \Sigma$, and dually. Hence, if $n = 1$, then the claim follows from fullness of Σ . Now, assume that the statement holds for $n - 1$. There exists $k \in \mathbb{Z}$ such that $\tau^k M_{n-1}^\bullet$ belongs to the same connected component of Σ as M^\bullet . By translation, there exists an irreducible morphism between $\tau^k M_{n-1}^\bullet$ and $\tau^k M_n^\bullet$. Another application of the case $n = 1$ gives the result. \square

Proposition 4.1.4. The subquiver Σ is a G -stable section in Γ .

Proof. First, Σ is a full subquiver of Γ by definition. Moreover, Σ contains no oriented cycles (since Γ is directed) and intersects each τ -orbit of Γ exactly once by 4.1.2. Since Σ is clearly convex, it remains to show that Σ is connected and G -stable. For the connectedness, assume that M^\bullet, N^\bullet are two objects in Σ . Since Γ is connected, there exists a walk from M^\bullet to N^\bullet in Γ . By 4.1.3, there exists $r \in \mathbb{Z}$ such that $\tau^r N^\bullet$ belongs to the same connected component of Σ as M^\bullet . Since Σ intersects each τ -orbit exactly once, we get $r = 0$, and so Σ is connected. Finally, Σ is G -stable since, for each $\sigma \in G$, the functor ${}^\sigma(-) : D^b(A) \longrightarrow D^b(A)$ commutes with the Auslander–Reiten translation τ by 2.1.3, and thus preserves the sectional paths. \square

Proposition 4.1.5. Let A be a piecewise hereditary algebra of type $\text{mod } H$, for some hereditary algebra H , and G be a torsion group. Then, for any action of G on A , there exist a hereditary algebra H' and an action of G on H' inducing a G -compatible equivalence of triangulated categories between $D^b(A)$ and $D^b(H')$.

Proof. Since G is a torsion group, it follows from 4.1.4 that $D^b(A)$ admits a G -stable section Σ . Let $H' = \text{End}_{D^b(A)} \Sigma$. By Rickard's Theorem [12], there exists an equivalence of triangulated categories $E : D^b(A) \longrightarrow D^b(H')$ which takes Σ to the full subquiver Ω of projective H' -modules in $D^b(H')$. Under the identification $D^b(A) \cong D^b(H')$, $D^b(H')$ is endowed with an action of G and, for any $\sigma \in G$, we let ${}^\sigma(-) : D^b(H') \longrightarrow D^b(H')$ denote the induced automorphism. These automorphisms restrict to automorphisms of $(\text{mod } H')[i]$, for any $i \in \mathbb{Z}$, by [5, (IV.5.1)]. To prove our claim, it then remains to show that there exists an action of G on H' such that the induced action on $D^b(H')$ (see Section 2.1) coincides with the action carried from $D^b(A)$.

For this sake, observe that since Σ is G -stable, so is Ω . Moreover, Ω is the ordinary quiver associated to H' , and so $H' \cong k\Omega$. We define an action of G on H' as follows: let $\{e_1, e_2, \dots, e_n\}$ be a complete set of primitive orthogonal idempotents of H' and let $\{P_1, P_2, \dots, P_n\}$ be the associated indecomposable projective H' -modules, each of them being a vertex of Ω . Then, for $\sigma \in G$, set $\sigma(e_i) = e_j$ if ${}^\sigma P_i = P_j$. Moreover, if α is an arrow of Ω , then set $\sigma(\alpha) = {}^\sigma \alpha$. This defines an action of G on H' , and further on $D^b(H')$. For each $\sigma \in G$, we let ${}^\sigma(-) : D^b(H') \longrightarrow D^b(H')$ denote the induced automorphism. The equivalences ${}^\sigma(-)$ and ${}^\sigma(-)$ coincide, up to a functorial isomorphism, because they clearly coincide on projectives. \square

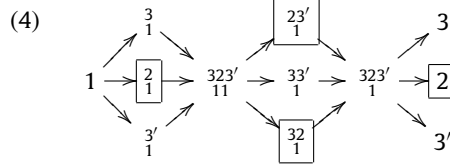
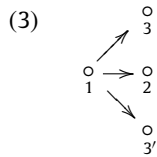
As we will see, the above proposition will play a major role in the proof of Theorem 2. We now consider the case where A is a piecewise hereditary algebra of type $\mathcal{H} = \text{coh } \mathbb{X}$, for some weighted projective line \mathbb{X} , and G is a group acting on A . For more details concerning the categories of coherent sheaves on a weighted projective lines, we refer to [3, 10].

Let p_1, p_2, \dots, p_r be a set of natural numbers and $\mathbb{X} = \mathbb{X}(p_1, p_2, \dots, p_r)$ be a weighted projective line over k of type p_1, p_2, \dots, p_r (in the sense of [3]). Let $\mathcal{H} = \text{coh } \mathbb{X}$ be the category of coherent sheaves on \mathbb{X} . Then \mathcal{H} is a hereditary abelian category with tilting objects. It is known that there exists a tilting object $T \in \mathcal{H}$ such that $\text{End}_{\mathcal{H}} T = C(p_1, p_2, \dots, p_r)$, where $C(p_1, p_2, \dots, p_r)$ is a canonical algebra of type p_1, p_2, \dots, p_r (in the sense of [13]). An important classification tool is the slope function $\mu : \mathcal{H} \longrightarrow \mathbb{Q} \cup \infty$; see [10].

Then, we get the following proposition, whose proof easily follows from [10, (4.4)]. We include a sketch of the proof for the convenience of the reader.

Proposition 4.1.6. Let A be a piecewise hereditary algebra of type $\text{coh } \mathbb{X}$, for some weighted projective line \mathbb{X} , and G be a group. For any action of G on A , there exist an action of G on $\text{coh } \mathbb{X}$ and a G -compatible equivalence of triangulated categories between $D^b(A)$ and $D^b(\text{coh } \mathbb{X})$.

and it is easily seen that if T is the direct sum of the identified indecomposable modules in the above diagram, then T is a G -invariant tilting H' -module such that $\text{End}_{H'} T \cong A$. Now, since G is cyclic, the method explained in [11, (Section 2.3)] gives that $H'[G]$ is (Morita equivalent to) the path algebra of the quiver (3) below, and the Auslander–Reiten quiver of $H'[G]$ is given by the quiver (4) below.



where the identified indecomposable modules in the quiver (4) correspond to the indecomposable direct summand of the tilting $H'[G]$ -module FT of 3.1.1. It is then easily verified, as predicted by 3.1.1, that $\text{End}_{H'[G]} FT \cong (\text{End}_{H'} T)[G] \cong A[G]$.

Acknowledgements

The authors thank Ibrahim Assem who suggested the problem considered in this paper. This work was started while the second author was visiting the Université de Sherbrooke and Bishop's University under a postdoctoral fellowship, and pursued while the first and third authors were consecutively visiting the Universidad de la República del Uruguay. They would like to express their gratitude to Marcelo for his warm hospitality. Finally, the third author thanks Helmut Lenzing for a brief but fruitful discussion while he was visiting the NTNU in Norway in January 2007.

The first author was partially supported by ISM of Québec. The second author was partially supported by CRM-ISM of Québec. The third author was partially supported by NSERC of Canada and FQRNT of Québec.

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